DATA SHEET

Return to index

Sheet #:	D3b.1
Title:	CURVES, SUPERELEVATION AND EASEMENTS
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Page:	1 of 20

CURVES ON A MODEL RAILROAD

Most model railroads are both blessed and cursed by an excess of curves. This is because, while the prototype tries to use the shortest and straightest routes, the modeler usually has to bend the track around basement corners and along benchwork that doesn't lend itself to long straight runs.

The blessing is that curves add character to what might otherwise be rather boring. The curse is that all of the civil engineering needed to prevent derailments on a prototype also applies to the model. So to get the benefits of curves without the derailments, modelers must use the same engineering methods used by the prototype railroads. Fortunately, there are simplifications that make the process easier. And, for those interested, the sheet provides the mathematical bases for the civil engineering.

A NOTE ABOUT SCALE

For space reasons, because of the large number of scales, this data sheet cannot provide conversions for each. So, most measurements will be given in ***prototype*** feet and inches. To convert to your scale, see Standard S-1 for the appropriate conversion factor. Or better, don't convert! Go out and buy a scale ruler for your scale, and use the scale measurements directly. How long is a 40 foot box car? 40 feet, of course. You can see already how much easier this is!

THE THREE TRACK TYPES

There are three types of track:

- ***Straight Track***, also called ***tangent*** *track*.
- ***Circular Curve***, so-named because it is always ***part of a circle***, with a ***constant radius.***
- ***Easement***, also called a ***transition curve***. An easement is curved track that does not have a constant radius, so it is not a circular curve. In fact, an easement has a radius that changes throughout its length. Easements are used to transition from straight track to circular curves, and also to transition between two circular curves that have different radii (a ***compound curve***).

CIRCULAR CURVES

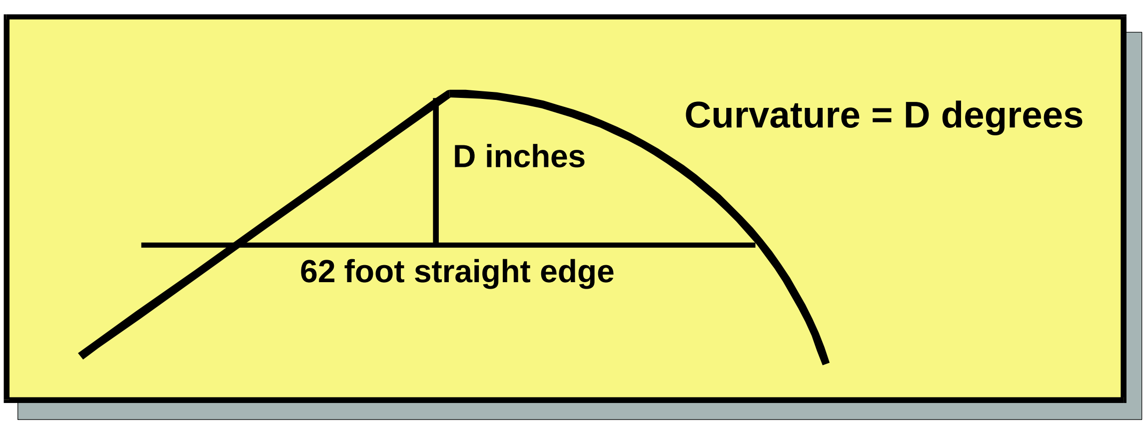
The most important characteristic of track is its ***curvature***, also called ***degree of curvature,*** which is a number that describes how sharply it turns. The curvature of straight track is zero. Circular curves have a constant curvature that is greater than zero and it is important to understand that this curvature is constant throughout the length of the curve. On the other hand, an easement has a curvature that changes throughout its length.

CIRCULAR CURVES - continued

Modelers often describe a curve by its ***radius*** and geometric ***center***. This is because the modeler can often easily view both the curve and its center. A civil engineer, however, expresses a curve's sharpness by its curvature because, when you're standing near a curved track in the prototype railroad, it is hard to visualize just where the center of that curve is. It is probably not accessible anyway, usually being in the middle of some obstacle like a swamp or mountain. In the prototype, the reason for the curved track in the first place was probably to get around just such an obstacle. So, from where the track was at any given time, the civil engineer would run out a 100 foot cord (chord) at an angle from the tangent equal to one half of the degree of curvature and mark the centerline there. Repeated every 100 feet, this would result in the desired circular curve. There are many other surveying techniques available, but most use the degree of curvature to make the technique simpler in the field.

Prototype curves of less than 5 degrees of curvature can be laid entirely with straight rail sections, whereas sharper curves require that the rail be curved on a rail bender before being spiked into place. Curves of less than 8 degrees of curvature are rigorously kept to the exact nominal gauge but sharper curves require that the gauge be widened to accommodate wheel flanges - the axles cannot individually swivel, being part of a rigid wheelbase like a truck, and therefore become skewed (angled) to the turning radius. When widening the track, only the inside rail is moved, away from the track centerline. Also see the notes in Standards S-3 and S-4 for track and wheel guidelines.

A rule of thumb useful for all curves up to 60 degrees is to place a 62 foot straight-edge (or string) against the inside of the outer rail and measure its maximum deflection from that rail (at the halfway point) in scale inches. That number is the degree of curvature. This method is accurate to one-tenth of a degree up to 20 degrees, to one degree up to 40 degrees and four degrees up to 60 degrees, regardless of gauge or scale. This trick, as illustrated below, is used on the prototype all the time.



CIRCULAR CURVES - continued

Another consideration of curves is coupler and diaphragm angle and, where equipment of different length is coupled, their different displacements that cause coupler misalignment. Always err on the side of gentle when planning and laying out your curves.

Table 1: CURVES

TYPE OF CURVE	DEGREE OF CURVATURE	RADIUS (ft)	EQUIPMENT CLASS (RP-11)	GAUGE WIDENING
Straight Track	0	infinite	all classes	none
Broad Curves (prototype and model)	3 6 8 10	1910 955 717 574	all classes all classes all classes all classes	none none .125 inch .250 inch
Moderate Curves (prototype)/ Broad Curves (model)	12 13 14 15 16 18 20	478 442 410 383 359 320 288	all classes all classes all classes all classes all classes all classes all classes	.375 inch .500 inch .625 inch .750 inch .750 inch .750 inch .750 inch
Sharp Curves (prototype)/ Broad Curves (model)	25 30	231 193	all but P all but O-P	.750 inch .750 inch
Sharp Curves (prototype)/ Moderate Curves (model)	35 40	166 146	all but N-P all but H, M-P	.750 inch .750 inch
Sharp Curves (prototype and model)	50 60	118 100	only A-E, F, J-K only A-E, J	.750 inch .750 inch
Interurban/ Street Cars	80 100 180	78 65 50 36	only A-D only A-C only A-B only A	.750 inch .750 inch .750 inch .750 inch

DOES A MODELER CARE ABOUT EASEMENTS AND SUPERELEVATION?

You bet! Modelers scale down the size, and the speed, of their models, but few know that when a model is in motion, its mass becomes a very important factor. A typical HO-scale model of a 150-ton passenger diesel should weigh less than 8 ounces, but most weigh twice that. The model is ***double*** the scale mass. The problem is not as bad in the larger scales, but much worse in the smaller ones. The bottom line: the considerations put into track planning by civil engineers are even more important to the modeler than to the full-sized railroads.

Easements and superelevation are not silver bullets, however. They will not correct faulty-running equipment and a kink will derail a train as fast as a flawed turnout. The best way to get the benefits of these techniques is to be ***gentle***. This means gentle curves, gentle easements and gentle approaches to superelevated track.

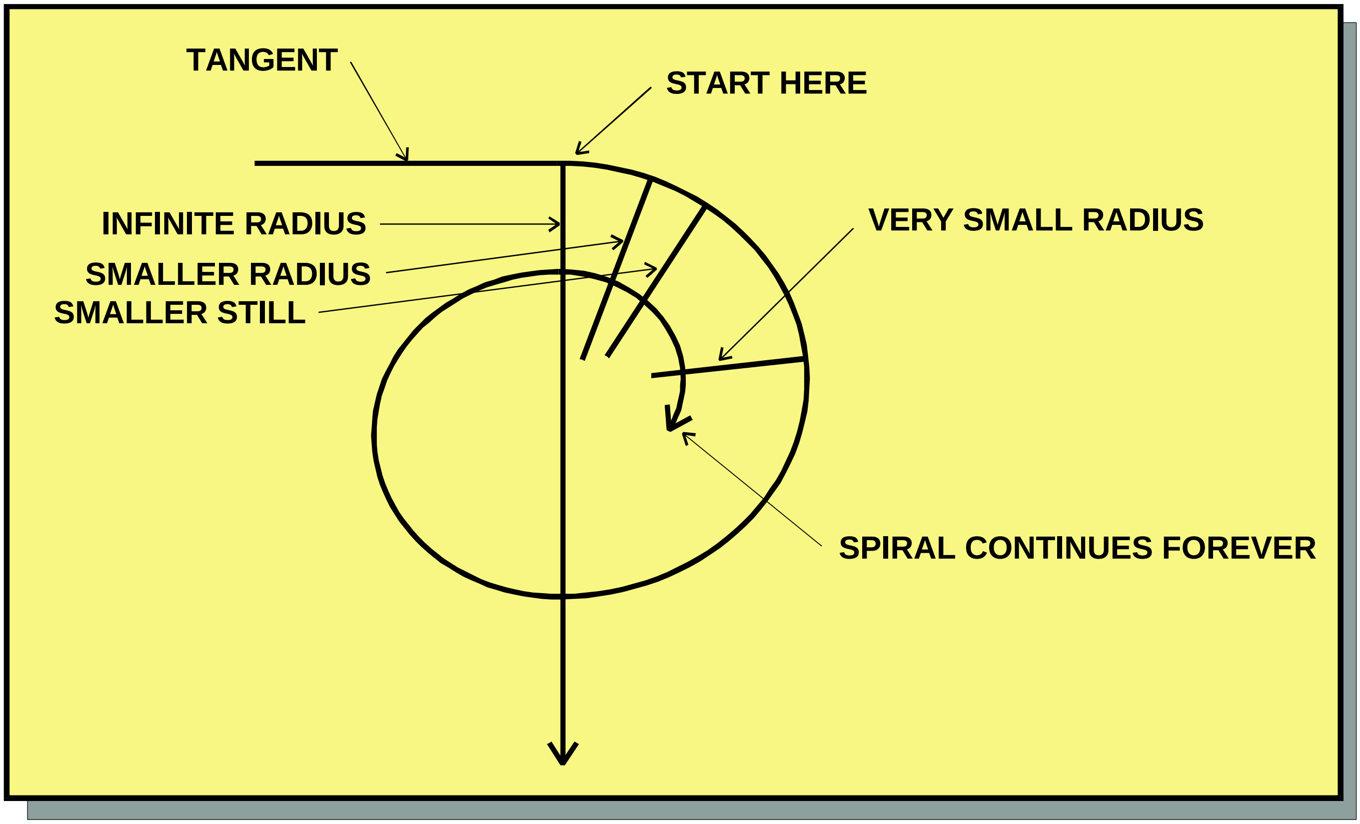
WHY HAVE EASEMENTS?

Imagine you are driving down the highway. Where the road is straight, look at your steering wheel - it should be centered. When you are in the midst of a large curve, look again - your steering wheel has turned by some appropriate angle. Take note of this "curve position." Now, go back to the beginning, drive down that straight section at 60 mph and, where the curve starts, instantly turn the steering wheel from straight to the "curve position." On a real highway, you would now be flipping over and over into the oncoming traffic.

This is why you need easements: they allow you to "gradually" turn that wheel, "easing" it to the "curve position." Prototype railroads use easements just like highways, and your model railroad should, too. They turn the trucks under your cars gradually, thus avoiding shock to cars and rail. They prevent derailments. You cannot just change from straight motion to circular motion, or between circular motions of different radii, without a lot of disagreeable force being generated that is just itching to push your train off the track.. This force is the same one that creates turbulence at the backs of airplanes and ships' hulls. A lot of research has been conducted to discover the reasons for this turbulence and to minimize it for the flyers and sailors - the same solution also applies to minimizing derailments for both the prototype and model railroader.

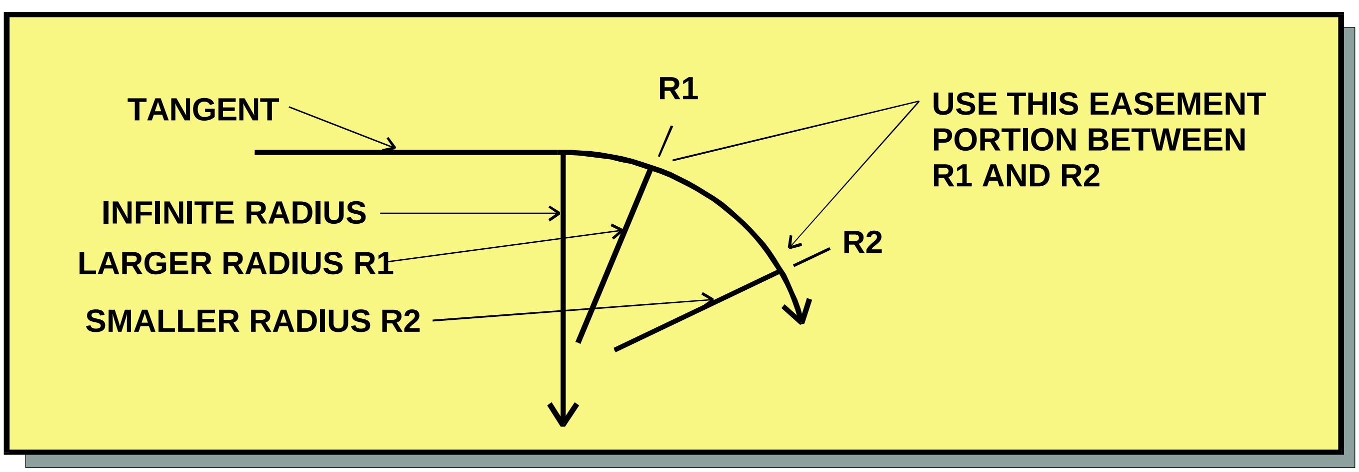
Turbulence is caused by abrupt curve changes i.e. when the change in curvature is not smooth. A curve that has only smooth curvature changes is called ***fair*** by engineers (hence the name ***fairing*** on airplane fuselages). There are infinitely-many types of fair curves but the one most often used in railroad engineering is the Cubic Spiral, which looks like the drawing on the next page.

WHY HAVE EASEMENTS - continued



The useful parts of the cubic spiral easement are those at or near where the spiral starts. You will rarely if ever see an easement that makes a full right angle (90 degree) turn.

In addition to easing from tangent track to a circular curve, you will often need to ease between two curves of differing curvature. We use the same easement shape, but omit the part close to the tangent i.e. we simply extract the part of the easement that has the appropriate curvatures at each end and insert it between the two curves, as illustrated below:



WHY HAVE EASEMENTS - continued

Prototype railroads use easements on all curves on Class A trackage and on all main tracks where the change of curvature is one-half degree or more. They also use easements wherever possible in yards and sidings. Generally, almost all prototype track is eased to some extent, either by being specifically engineered, or by eye during the actual track-laying.

Since an easement contains a varying curvature throughout its length, any gauge-widening needed is smoothly adjusted along the easement.

WHY SUPERELEVATE?

Superelevation is simply the railroad's equivalent of banking a curve on a highway, with which we are all probably familiar. On a curve, a train tends to lean "away" from the curve center due to centrifugal force. The flanges keep the wheels on the track but the top of the train wants to keep going straight, giving the train a tendency to flip. Superelevation also increases the comfort of passengers and crew.

To reduce this twisting action, the prototype railroad raises the outside rail on the curve, keeping the inside rail at grade - hence the term "super"-elevation. On a model railroad, superelevation slightly reduces the chance of flipping over, but its main benefit is that it just plain looks spectacular.

The perfect amount of superelevation is based on the curvature and speed. For example, a 4 degree curve taken at 60 mph requires a superelevation of 9.5 inches. However, standard railroad practice is to never use a superelevation greater than 8 inches. Also, where trackage must accomodate both slow and fast traffic, standard railroad practice is to compute the superelevation for the fast traffic and then reduce it by 3 inches, to obtain what is called "unbalanced superelevation." Standard railroad practice is to never use a superelevation for fast and slow trackage greater than 6 inches.

Modelers can typically use 6 inches of superelevation. Incidently, this amount of superelevation results in an almost 11% grade sideways across the rails.

The prototype lowers the speed limits in cases where the amount of superelevation is less than optimum, as shown in Table 2. Because our model curves are often far sharper than the prototype would use in the same circumstances, lower mainline speeds would be a prototypical alternative to broader curves or impractical superelevation.

Table 2: TYPICAL STANDARD GAUGE SPEED LIMITS (mph) FOR NON-OPTIMAL SUPERELEVATION

DEGREE OF CURVE	INCHES OF SUPERELEVATION 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
0.5	40	55	65	75	85	95	102	110	------------NO LIMIT-----------			
1.0	25	37	45	55	60	67	72	77	82	87	90	95
1.5	20	30	37	45	50	55	58	62	65	70	74	77
2.0	20	25	32	37	42	47	52	55	59	62	65	68
2.5	20	25	30	34	37	42	45	49	52	55	58	60
3.0	18	22	27	32	35	39	42	45	48	50	53	55
3.5	18	22	25	29	32	35	39	42	44	47	49	52
4.0	17	20	24	27	30	34	37	39	41	43	45	47
4.5	17	19	22	25	29	32	34	37	39	41	43	45
5.0	17	19	22	25	28	30	33	35	37	39	41	43
5.5	16	18	20	24	27	29	31	33	35	37	39	41
6.0	15	17	19	22	25	28	30	31	33	35	37	38
7.0	14	16	18	20	23	25	27	29	31	32	34	36
8.0	13	15	17	19	22	24	26	27	29	31	32	34
9.0	12	14	16	18	20	22	24	26	27	29	30	32
10.0	10	13	15	17	19	21	22	24	26	27	28	30
11.0	10	12	15	17	19	20	21	23	25	26	27	28
12.0	10	12	14	16	18	20	21	22	23	25	26	27

RUN-OFF

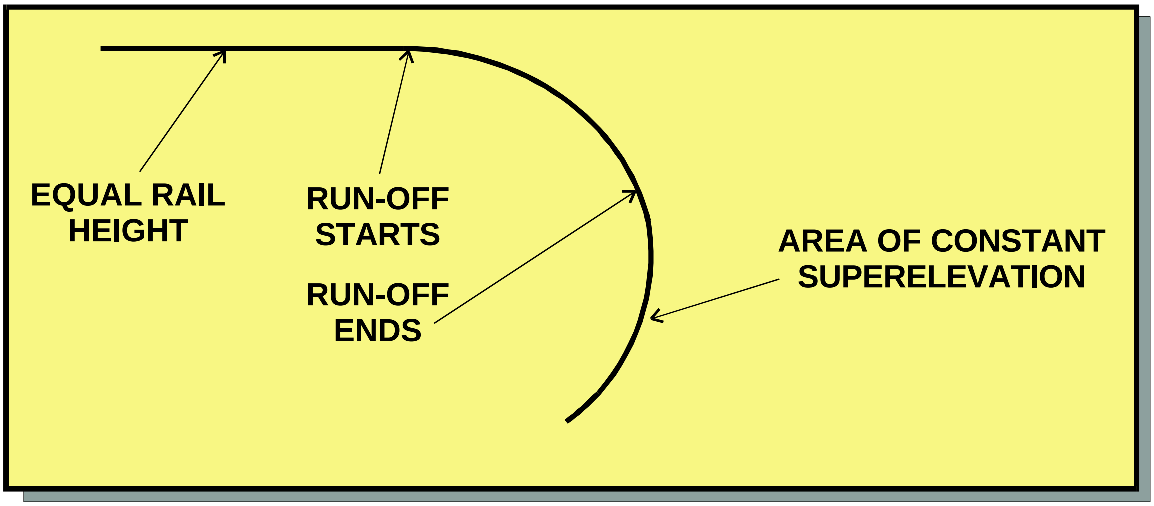
Superelevation is maintained throughout the area of constant curvature, but must be gradually built up on the approaches. This change in superelevation is called ***run-off***. Insufficient run-off is another source of derailments. The run-off must be long enough, i.e. gentle enough, to prevent the lifting of the wheel flanges over the rails. Longer run-off is also preferred in the prototype since the twisting motion causes discomfort to passengers and crew.

Prototype railroads usually use at least 70 feet of run-off for each inch of superelevation. A 6-inch superelevation therefore requires a run-off extending 420 feet - much too long to be feasible on most model railroads. It is suggested that the run-off on a model railroad be at least 100 scale feet, but extending to 200 scale feet would be much, much better.

In general, the minimum run-off is less than the minimum easement, so the run-off can be located in the easement. Most prototype railroads start the run-off at the same spot as the easement so that the run-off is more gentle than the minimum.

RUN-OFF - continued

In the model, run-off must start and stop with a gentle vertical curve to prevent an abrupt change in elevation that raises a wheel off the rail. Pay particular attention to how your trucks negotiate a run-off before deciding how much superelevation and run-off to use. You may find that adjusting truck-screw tension is critical. Also, cars with a high or off-center center-of-gravity may make high superelevation impractical.



TRACK WITHOUT EASEMENTS OR SUPERELEVATION

Is there any? Yes. Look at RP-12. Theoretically, there are only straight and circular curved rail sections in a turnout, ie they have no easements. Superelevation is rarely possible in a turnout since the frog is both on the inside rail of one track and the outside of the other. The prototype compensates for this by restricting the speed through turnouts. See Data Sheet D8a.1 for typical prototype speed limits in turnouts. As speed rises above those limits, the chances of a derailment increase dramatically.

Prototype railroads rarely use superelevation in crossings, for the same reason as with turnouts.

The prototype rarely if ever uses superelevation without easements. This situation is best avoided in the model, as well.

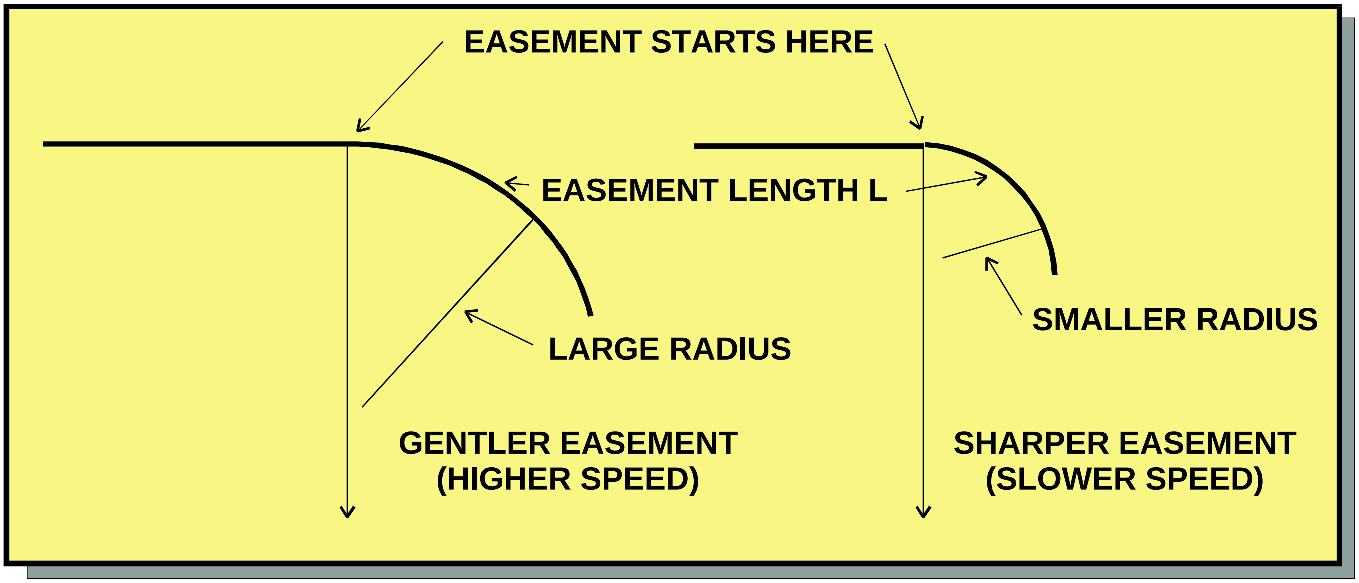
It is not practical to superelevate streetcar and interurban tracks laid in paved streets. In those areas, the lack of superelevation is made up for by using reduced speeds.

EASEMENT RATES

Easements can be anywhere from gentle to fast, depending on the type of equipment you run on that trackage, and the maximum speed. On the next page you can see two easements, one with a sharper turn. The sharper turn is due to a faster easement rate, i.e. where the easement must effect a larger change of curvature in the same length (as illustrated), or where the same curvature

EASEMENT RATES - continued

change must be accomplished in a shorter length. The gentler easement will permit longer equipment and higher train speeds.



The prototype uses various guidelines for the easement rate. In typical prototype practice, the ***minimum easement length*** on Class A trackage is not less than 100 feet, and preferably 150 feet, for each inch of superelevation. On other classes of track, the minimum easement length is not less than 60 feet, and preferably 100 feet, for each inch of superelevation. To maximize comfort and safety and minimize wear-and-tear on the equipment, the prototype will use the longest easement it can. Note that the minimum easement length is usually longer than the minimum ***run-off*** on a typical curve, so sometimes the prototype track will start to ease into the curve before the run-off begins.

A particularly bothersome problem for modelers is that, since we generally use sharper curves than the prototype, any misalignment in couplers or diaphragms can cause derailments. Not only must couplers and diaphragms permit a sharp coupling angle but equipment of differing lengths, which will extend different distances away from the track centerline, may not be able to negotiate the curve. Furthermore, where track changes curvature, neighboring pieces of rolling stock are on different curvatures, so even equipment that is the same length may misalign and derail. The answer is to use the most gentle easement you can.

Easements look a little bit like the curved part of a turnout. The match is not perfect, because turnouts are straight through the frog (see RP-12), whereas an easement just gets curvier. But, the general idea of describing easements using the same numbering system as for turnouts is appealing. A #8 easement behaves much like the diverging route of a #8 turnout.

Be generous with your easements. If you use #8 turnouts on your mainline, use #8 or larger easements.

WHAT ABOUT SECTIONAL TRACK?

Sectional-track manufacturers do not, at the time of writing, make easement sections. Sectional-track users must either use flexible track for their easements or approximate easements by using large radii sections between straight and sharper-radius sections. This technique is useful from small scales right up to large-scale garden railroads where the rail may be hard to bend.

Superelevation presents a huge problem to sectional track users because, when you raise the outer edge of a section, its ends descend, making both rails arc vertically. Unless you can bend the rails to level out this arc, sectional track cannot be superelevated.

TRACKPLANNING? START HERE!

1. Decide on what kinds of traffic you will have on your railroad: fast passenger trains, fast freights, slow freights, trolleys, RDCs, etc. Consult RP-11 to see how the various equipment classes affect your track plan. Each train will be governed by its most restrictive equipment class.

2. Divide your railroad into trackage classes: mainline, interchange, classification yard, industrial switching, branchline, staging, etc. Each class is characterized by the kinds of traffic you will run on it and the maximum speed. (You can cheat a little by running trains on underclassed track if you run them slowly). You may have more than one mainline class, one with restricted speeds and one where you can really open the throttle. You may have areas restricted to freight trains and other areas restricted to passenger trains.

3. For each class of track, you will need to assign three critical values:
- maximum curvature (minimum radius) (see RP-11);
- maximum easement rate, using the guidelines in this data sheet, but probably the same or one higher number than your minimum turnout number;
- maximum superelevation, which will probably be either 0 or 6 scale inches.

You will end up with a table like this:

TRACKAGE CLASS	MAXIMUM SPEED (mph)	MAXIMUM CURVATURE (degrees)	MINIMUM EASEMENT RATE	SUPERELEVATION (inches)
Freight classification yard	20	30	#4	0
Passenger classification yard	20	25	#6	0
Mainline - no speed restrictions	90	20	#10	6
Mainline - with turnout or crossover	40	20	#10	0
Mainline - slow speed	30	25	#6	6
Industrial trackage etc.	20	35	#4	0

TRACKPLANNING - continued

4. Decide where your straight and maximum-curvature track absolutely ***must*** be, based on benchwork/roadbed restrictions or other obstacles such as the washing machine.

5. Sketch out the track between these restrictions. You will see these different track scenarios:

a) Where straight track changes to the maximum curvature, use the full easement from the graph at the line corresponding to that curvature.

b) Where straight track changes to less than maximum curvature (ie. is less sharp than your maximum curvature), use the easement from the graph that corresponds to the lesser curvature, or consider using a sharper curve and a longer easement. This will permit a higher superelevation.

c) Where straight track connects to a different straight track, i.e. that will require a curve of some sort, measure the total angle of turn. Consult the graph to determine what angle the appropriate easement will turn to where it curves at your maximum curvature. You will need two easements and one curve in between. Try to use the longest easement and the shortest, sharpest curve. This will permit the highest superelevation.

d) Where curved track changes to curved track with a different radius, use the difference in easements from the graph. There is a nifty tool referred to in the construction section (later) that is very handy with compound curves.

e) Where curved track changes to curved track with the opposite turn (ie. a reverse or S-curve), use the full easement from the graph to straighten out the track, and then another easement to ease into the other curve.

6. Determine what superelevation you need in each circular curved section. Often, the object of the game is to have the track that is of constant curvature to also be of constant superelevation. Decide where each run-off starts. Always use the longest run-off possible. Since the minimum easement will almost always be longer than the minimum run-off, most run-offs can start where the easement starts to good effect.

THE EASEMENT GRAPH

The easment graphs are shown in Data Sheet D3b.2. The graph shows representative easement rates from #3 to #12. The easement lines are track centerlines. The rectangular grid shows the distance from the start of the easement, both along the tangent and offset from the tangent. Two other line types are superimposed on the easements. One is a representative set of angles turned by the easement, and the other is a representative set of curvatures and radii. You can readily see how these values interrelate. The second graph shows an enlargement of the area close to where the easement starts.

A NOTE ABOUT GAUGE

The graphs are for standard gauge (4'-8½"). If you use another gauge, you can use the same graphs, but you must relabel them. First, compute your gauge factor. For example, the gauge factor for 3-foot gauge is 3' / 4'-8 ½" = 0.637. The easement rates still correspond to the turnout numbers in your gauge.

The grid lines are expressed in feet. Multiply the grid numbers by your gauge factor. For example, change the 10-foot grid line to 6.37 feet.

The radius figures must similarly be multiplied by your gauge factor. Then compute the curvature figures using formula 2 or formula 3 below.

SPECIAL CASES, COMPROMISES AND RULES-OF-THUMB

In cases where the optimal superelevation or easement can't be achieved, railroads compromise. There are numerous rules-of-thumb. For example, it is better to have a constant, lower superelevation on a curve than to have a higher superelevation or an uneven run-off approaching that curve. Also, it is better to have a shorter, sharper circular curve than to have a short easement.

Where there is not enough room for the minimum easement at both ends of a curve, preference is given to the direction of the faster traffic - that easement is not shortened below the minimum length.

If you cannot have enough run-off, reduce the height of the superelevation and run your trains slower. The run-off should never be less than the minimum. If the run-off is less than the easement length, the run-off extends smoothly into the tangent. If the tangents are too short, the prototype reduces the superelevation to maximize the smoothness of the run-off.

Where a compound curve is separated by an easement, all of the run-off occurs in the easement, as uniformly as possible. Where a compound curve is not separated by an easement, the superelevation is constant in the sharper curve and the run-off is put into the broader curve.

On reverse curves with easements but no tangent track between them, the run-off must uniformly occur in each easement so that they meet with the rails level. If there is a tangent track but no easements, the rails must be level at a point proportionately along the tangent corresponding to the relative superelevations, to minimize changes in height. Where there is not enough length in the tangent and easements, the run-off extends into the circular curve at the maximum rate permitted for the class of trackage.

Where two tracks run parallel in a curve, the superelevation of the outer track must be less than that of the inner track or the tracks must be separated by an extra amount to prevent sideswiping. Modelers usually increase the track separation a few scale feet. Also, since the inner track has a sharper curvature, its easement is either longer or faster. Usually it uses the same easement rate and starts easing earlier in the tangent.

SPECIAL CASES, COMPROMISES AND RULES-OF-THUMB - continued

On trackage where the speed is never greater than ten mph, like sharp curves in yards and sidings, there is no superelevation.

Any superelevation other than the *perfect* amount results in uneven rail wear. If the rail wear is too uneven, rather than replace the rail too often, the prototype railroads adjust the superelevation to a better value, or reduce the speed of faster trains.

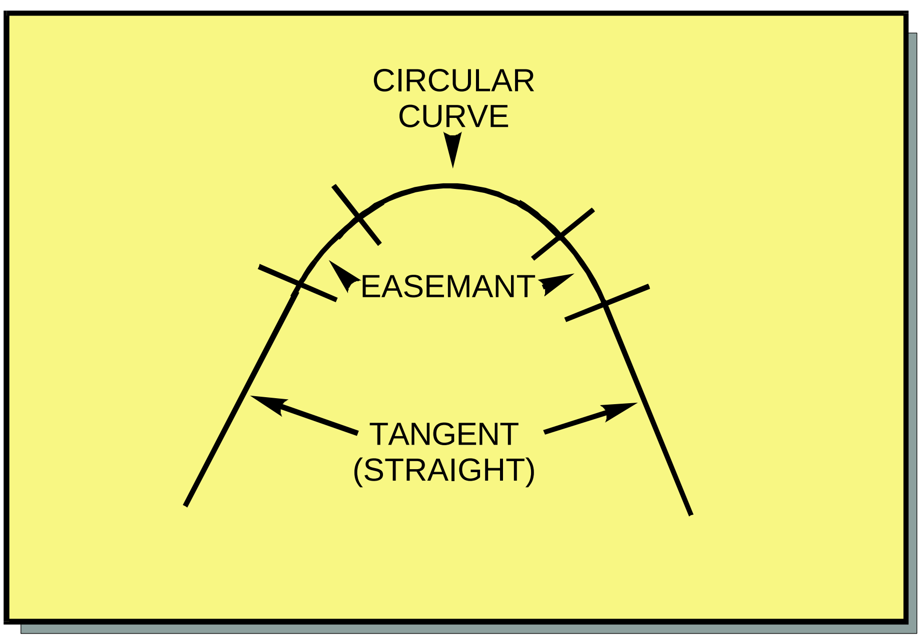
In a reverse curve, or S-curve, if you have no easements, ensure that there is a tangent track between the curves at least as long as the maximum car length for that trackage. Easements naturally prevent S-curves.

With dual-gauge trackwork, use the easement for the larger gauge.

Example One: A 90-degree Turn

You are trying to build an inside or outside corner on your mainline. The mainline is straight at each end of the curve, and turns 90 degrees in between. Your layout standard calls for a maximum mainline curvature of 30 degrees (193 feet). You want to use the maximum superelevation possible.

You will need two easements here, one at each end of the curve, between the straight track sections and the circular curve. They are mirror images of each other, so you only need to perform one set of calculations. The way to think of this situation is that you start with straight track, turn a bit on an easement, turn a bit more on a circular curve with a curvature of 30 degrees, finish the turn on another easement, and end up straight again, as illustrated below:



EXAMPLES - continued

The goal here is to select the appropriate easement rate so that your curve looks nice and performs well. Let's start by selecting a #6 easement. You know that you want to end up with a curvature of 30 degrees. Consult the graph. Locate the spot on the graph where the 30-degree curvature line crosses the #6 easement line. Here, you can read that the easement turn angle is approximately 16 degrees. This means that your 90-degree turn would be made up of 16 degrees of easement, followed by 58 degrees of circular curve, followed by 16 degrees of easement that gets you back to straight track. A bit of computation shows that this 58 degrees of circular curve (which is the same as a radius of 193 feet) amounts to 195 feet of superelevated track. This means that 3 or 4 boxcars or 2 or 3 passenger cars will all be at full superelevation together when going around this curve. This curve will look nice.

For comparison, let's look at the #8 easement. It is more gentle and turns approximately 39 degrees before it reaches your target mainline curvature of 30 degrees. So, your 90-degree turn would be made up of 39 degrees of easement, followed by 12 degrees of circular curve, followed by 39 degrees of easement that gets you back to straight track. This is probably overeased for your situation, since that 12 degrees of circular curve (radius 193 feet) is only 40 feet long. Your superelevation would look contrived and not quite *right*.

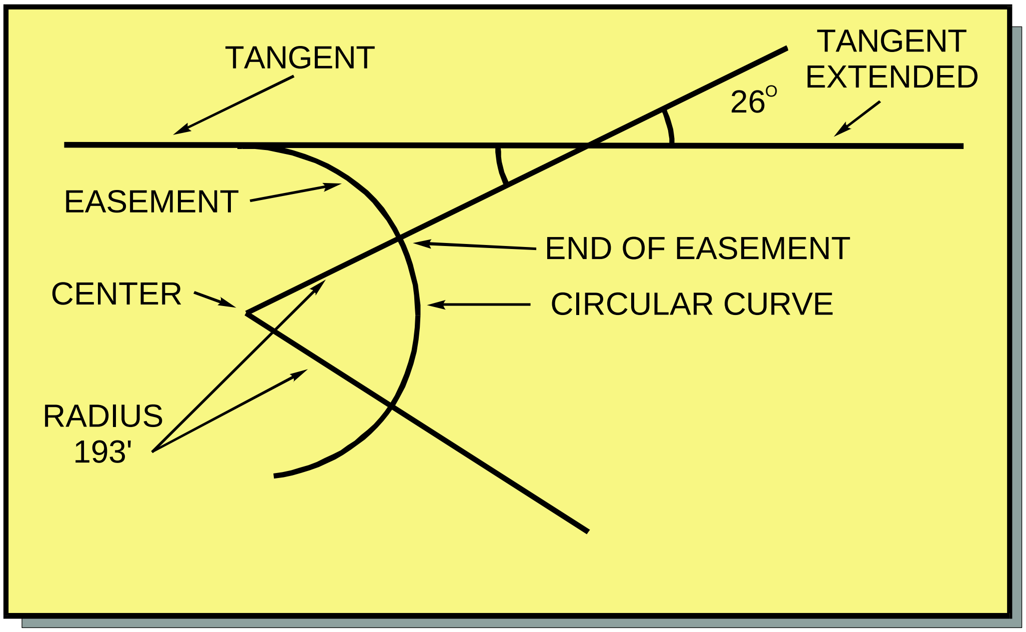
In between these two is the #7 easement, which turns approximately 26 degrees before reaching the target curvature. In this case your 90-degree turn would be made up of 26 degrees of easement, followed by 38 degrees of circular curve, followed by 26 degrees of easement that gets you back to straight track. This 38 degrees of circular curve (radius 193 feet) amounts to 128 feet of superelevated track, which is 2 or 3 boxcars but only 1 or 2 passenger cars. This curve may look fine, but not as nice as the longer one possible with the #6 easement. On the other hand, the #7 is more gentle, permitting higher speeds, and has a longer run-off, so the superelevation doesn't look as artificial. It is probably a better choice. Or, you could use something between #6 and #7. It is important, however, that you use a cubic-spiral easement ***shape*** so, unless you have some means of computing easements yourself, it is best that you stick to the ones drawn on the graph.

Speaking of run-off, how long is it? In all easements, the length of the easement is twice that of the circular arc it replaces. The #7 easement turns approximately 26 degrees, which in a 193-foot radius circle would be 88 feet of arc. The easement would be twice this, or 176 feet long. In the prototype, this would permit a superelevation of only 3 inches or so. If you start the run-off very gradually at each end (i.e. at both the tangent end and the circular curve end), you should be able to use a full 6 inch superelevation on this curve, for dramatic visual effect.

To help you lay out your easement, you can read the plotting coordinates from the top and side axes of the graph. In our example, the #7 easement ends approximately 172 feet along the tangent extended and offset from it approximately 26 feet. These two distances allow you to mark where the easement changes to circular curve. You can use other values from the graph along the #7 easement line to help locate the easement track centerlines.

With a bit of protractor work you can locate the center of the circular curve. It is 193 feet from the end of the easement, at an angle of 26 degrees from the tangent extended, as shown in the drawing on the next page:

EXAMPLES - continued



Example Two: A 180-degree Turn

In this example, we modify Example One to turn a half-circle, like at the end of a peninsula. In this case, the #8 easement is quite suitable since it leaves 102 degrees of circular curve. This is plenty to show off your superelevation. Each easement is 263 feet long, which permits a full 6-inch superelevation with a gentle run-off.

Example Three: A Compound Curve

You are trying to ease from a broad 20-degree curve (288-foot radius) to a moderate 35-degree curve (166-foot radius) in a mountainous area on a branch line. This is probably a case for a #6 or gentler easement.

This is the hardest easement problem to solve. Basically, you are not dealing with a full easement - i.e. one that starts at straight track and eases you into a circular cuve - but just a portion of an easement. This is because some of that curvature already exists in your track, in the 20-degree curve. So, you only need that part of the easement that gets you from there to a 35-degree curvature.

The hard part is getting the easement to line up with your two circular curves. It is important that each curve connects to the easement at the correct angle. Usually one of the curves is rather immovable so you must jockey the easement and the other curve around it until the angles line up.

One way is to make an easement template for your situation. Consulting the #6 easement line on

EXAMPLES - continued

the graph, estimate where the 35-degree curvature point is - it would be midway between the 30- and 40-degree curve lines. Then, mark on the same easement line a second point, at the 20-degree curvature point. It is the easement between these two points that you will use on your layout. Make a template in your track-planning scale by plotting that section of the easement.

Assume that the sharper curve is immovable. Tack a 20-degree curve to the 20-degree end of the easement, making sure that the track meets at the correct angle without a kink. Then find a place on the 35-degree curve where you can line up the 35-degree end of the easement and the 20-degree curve falls where you want it to be. You may have to try different easement rates to get the look you want.

If the 20-degree curve is the one that is immovable, the process is opposite to that above. Tack a 35-degree curve to the 35-degree end of the easement and then try to find a place on the 20-degree curve where the 35-degree curve falls where you want it to be.

There is a nifty tool mentioned above in the track-planning section that makes compound curves easier to lay out.

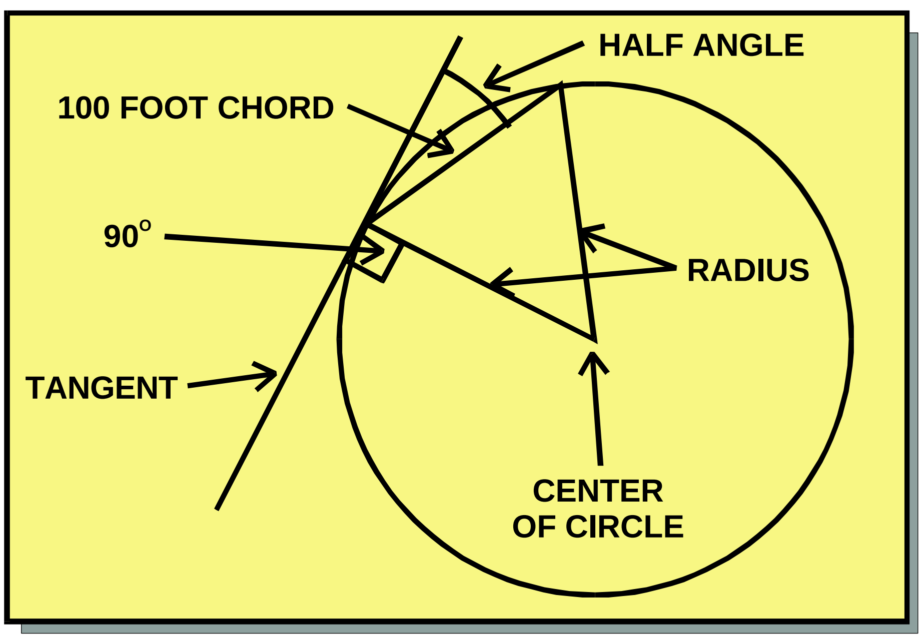
THE MATHEMATICS

The sections that follow are for those interested in a bit of the civil engineering that underlies prototype railroad practice.

THE MATHEMATICS OF CIRCULAR CURVED TRACK

Definition: **Degree of Curvature**: the sharpness of a curve, measured by the number of degrees of turn in a unit distance, usually degrees per 100 feet. It is also measured as twice the angle that a 100 foot chord on the curve diverges from the tangent.

Definition: **Tangent**: the centerline of straight track, or where the centerline would be if the track were extended straight from any point along an existing centerline.



THE MATHEMATICS OF CIRCULAR CURVED TRACK - continued

$$\text{Radius (ft)} = \frac{50\text{ ft}}{\sin(\text{Angle} / 2)} \qquad \text{(Formula 1)}$$

Angle (degrees of Curvature) = 2 arcsin (50 / Radius) (Formula 2)

$$\text{Angle (degrees of Curvature)} = \frac{5730}{\text{Radius (ft)}} \quad \text{(approximately)} \quad \text{(Formula 3)}$$

$$\text{Radius (ft)} = \frac{5730}{\text{Angle (degrees of curvature)}} \quad \text{(approximately)} \quad \text{(Formula 4)}$$

THE MATHEMATICS OF SUPERELEVATION

The formula for superelevation is based on the expected train velocity V (mph):

$$\text{inches of superelevation} = \frac{\text{gauge(inches)} \times V^2}{15 \times \text{radius(feet)}} \qquad \text{(Formula 5)}$$

For standard gauge, 4'-8½", and using the approximate degree of curvature, D = 5730 / r, the formula is:

$$\text{inches of superelevation} = 0.00066 \times D \times V^2 \qquad \text{(Formula 6)}$$

where D is the degree of curvature and V is the velocity in mph.

THE MATHEMATICS OF RUN-OFF

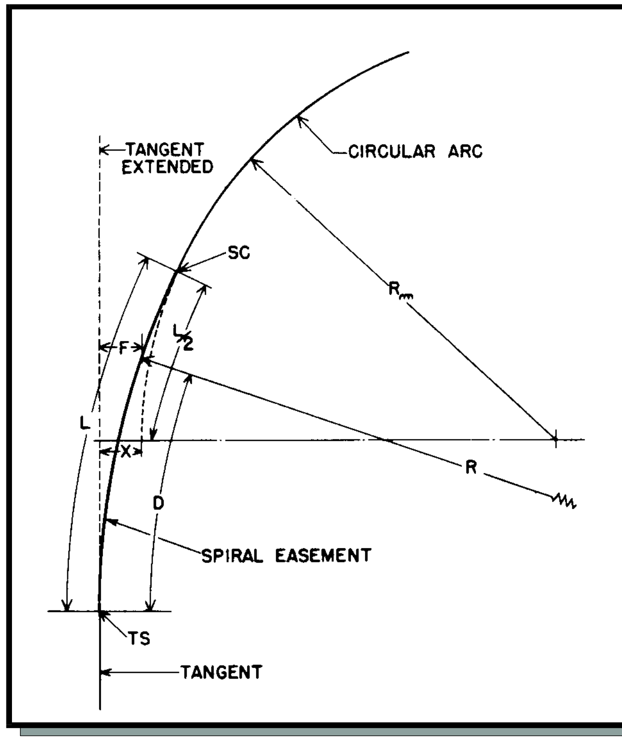
Experimentation showed that the limit is about 1.25 degrees of twist per second. In standard gauge, this equates to a maximum of 1.25 inches of superelevation per second of travel. At 60 mph the train covers 88 feet in a second, so the run-off should be no faster than 1.25 inches every 88 feet. A 60-mph 6-inch superelevation therefore requires a run-off extending over 350 feet.

$$\text{run-off length (ft)} = \frac{1.467 \times \text{speed (mph)} \times \text{superelevation (inches)}}{\text{track gauge (inches)} \times \tan(1.25\text{ degrees})} \qquad \text{(formula 7)}$$

run-off length (standard gauge) (ft) = 1.173 x speed (mph) x superelevation (inches) (formula 8)

THE MATHEMATICS OF EASEMENTS

TS - end of tangent, start of spiral easement
SC - end of spiral easement, start of constant curvature trackage
L - total length of spiral easement (feet)
Rm - radius of constant curvature trackage (feet)
D - length of a portion of the spiral easement (feet)
E - distance along tangent adjacent to easement of length D (feet)
F - offset from tangent to end of easement of length D (feet)
R - instantaneous radius after easement of length D (feet)
A - instantaneous angle of easement from tangent (degrees)
Ar - A expressed in radians
X - offset from tangent of where tangent would be if constant curvature trackage replaced the easement(ft)
G - track gauge (feet)
Af - angle at which outside rail crosses inside rail of tangent extended (radians)
Afd - Af expressed in degrees
K - easement factor (gaugeless or gauge=1 foot)
Kg - easement factor (your gauge)
N - easement rate (similar to frog number)



The following relationships characterize an easement:

$Af = 2 \arcsin(\frac{1}{2} N)$ (formula 9)

$K = 1.125 (1 + \cos Af)^2 / Af^3$ (approximately) (formula 10)

$Kg = K G^2$ (formula 11)

$Kg = Rm L$ (formula 12)

$X = Kg^2 / 48 Rm^3$ (approximately) (formula 13)

At any point along the easement, i.e. for any D, R and A, the following relationships hold:

$D = 2 R Ar$ (formula 14)

$Ar = D^2 / 2 Kg$ (formula 15)

$Kg = R D$ (formula 16)

THE MATHEMATICS OF EASEMENTS - continued

$F = D^3 / 6\ Kg$ (approximately) (formula 17)

$$F = D\left(\frac{Ar}{3} - \frac{Ar^3}{42} + \frac{Ar^5}{1320} - \frac{Ar^7}{75600} + \frac{Ar^9}{6894720}\right)$$ (better approximation) (formula 18)

$$E = D\left(1 - \frac{Ar^2}{10} + \frac{Ar^4}{216} - \frac{Ar^6}{8360} + \frac{Ar^8}{685440}\right)$$ (approximately) (formula 19)

Table 3: EASEMENT RATES

EASEMENT #	Afd (degrees)	K	Kg (standard gauge)	Kg (3 foot gauge)
3	18.9	118.2	2621	1064
4	14.3	283.6	6286	2552
5	11.4	556.9	12346	5012
6	9.5	965.3	21399	8688
8	7.2	2295	50877	20655
10	5.7	4489	99509	40399
12	4.8	7763	172083	89863

HOW TO CONSTRUCT CURVES AND EASEMENTS

See Data Sheets D3b.3.

For those interested in constructing a nifty tool that aids in laying out easements and locating the corresponding track centers, see the excellent article in the NMRA Bulletin December 1980 (check the Bulletin for reprint information or get a reprint from the NMRA's Kalmbach Memorial Library).

HOW TO CONSTRUCT SUPERELEVATION AND RUN-OUT

See Data Sheet D9i for the roadbed profile. Prototype railroads superelevate by raising the ballast, **not** by sloping the subroaded. This method is also applicable to garden railroads and other large scales. Other modelers either raise and sculpt the subroadbed under the ties to obtain the required roadbed slope, or use beveled stripwood or layers of tape under the ties.

REFERENCES AND CREDITS

NMRA Data Sheet D3c (1952): Hazen, Houghton, Ravenscroft and Shultz.
Hickerson, Thomas F.: Route Location and Design, Fifth Edition (1964), McGraw-Hill Book Company, New York.
Various Railroads: Maintenance of Way - Rules and Instructions.
Lukesh, John: Transition Curves, NMRA Bulletin December 1980, pp20-26.
Bill Sharpe.

RELATED DOCUMENTS

RP11 - Curvature and Rolling Stock
D1j - Grades and Angles
D3a - Properties of Curves
D3e - Model Railroad Roadbed
D5g - Determination of Minimum Radius, Rolling Stock
D8b - Train Speeds
D8e.2 - Realistic Operation - Prototype Freight Trains
D9i - Roadbed